Learn to Select Good Examples with Reinforcement Learning for Semi-structured Mathematical Reasoning

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Abstract

Recent large pre-trained language models such as GPT-3 have achieved remarkable progress on mathematical reasoning tasks written in text form, such as math word problems (MWP). However, it is unknown if models can handle more complex problems that involve heterogeneous information, such as tabular data. To fill the gap, we present Tabular Math Word Problems (TABMWP), a new dataset containing 38,431 open-domain problems that require mathematical reasoning on both textual and tabular data, where each question is aligned with a tabular context. We evaluate different pre-trained models on TABMWP, including the GPT-3 model in a few-shot setting. As earlier studies suggest, since few-shot GPT-3 relies on the selection of in-context examples, its performance is unstable and can degrade to near chance. This issue is more severe when handling complex problems like TABMWP. To mitigate this, we further propose a novel approach, PROMPTPG, which utilizes policy gradient to learn to select good in-context examples from a small amount of training data. Experimental results show that our method outperforms the best baseline by 5.31% in accuracy and reduces the prediction variance significantly compared to random selection. [1]

1 Introduction

Solving math word problems (MWPs) is a well-defined task to diagnose the ability of intelligent systems to perform mathematical reasoning as humans. However, most existing MWP datasets focus on textual math word problems only (Upadhyay & Chang, 2017; Amini et al., 2019; Miao et al., 2020; Cobbe et al., 2021). Tables, widely distributed in different documents contain rich structured information different from unstructured text. Solving MWPs in a tabular context is much more challenging than existing benchmarks since the system needs to make cell selections and align heterogeneous information before performing further numerical reasoning.

To fill this gap, we propose Tabular Math Word Problems (TABMWP), a new large-scale dataset that contains 38,431 math word problems with tabular context. There are two question types: *free-text* questions in which the answer is an integer or decimal number, and *multi-choice* questions where the answer is a text span chosen from option candidates. Different from existing MWP datasets, each problem in TABMWP is accompanied by a tabular context, which is represented in three formats: an image, a semi-structured text, and a structured table. Each problem is also annotated with a detailed solution that reveals the multi-step reasoning steps to ensure full explainability.

We first build a strong baseline using few-shot GPT-3 on TABMWP. A few in-context examples are randomly selected from the training set, along with the test example, and are constructed as a prompt for GPT-3 to predict the answer. However, recent studies have shown that this type of few-shot

[1]The data and code are available at `https://promptpg.github.io`.
Work was partially done while Pan Lu being an intern at AI2.

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square beads	\$2.97 per kilogram
oval beads	\$3.41 per kilogram
flower-shaped beads	\$2.18 per kilogram
star-shaped beads	\$1.95 per kilogram
heart-shaped beads	\$1.52 per kilogram
spherical beads	\$3.42 per kilogram
rectangular beads	\$1.97 per kilogram

Question: If Tracy buys 5 kilograms of spherical beads, 4 kilograms of star-shaped beads, and 3 kilograms of flower-shaped beads, how much will she spend? (unit: \$)
Answer: **31.44**
Solution:
Find the cost of the spherical beads. Multiply: \$3.42 × 5 = \$17.10.
Find the cost of the star-shaped beads. Multiply: \$1.95 × 4 = \$7.80.
Find the cost of the flower-shaped beads. Multiply: \$2.18 × 3 = \$6.54.
Now find the total cost by adding: \$17.10 + \$7.80 + \$6.54 = \$**31.44**.
She will spend \$**31.44**.

Sandwich sales		
Shop	**Tuna**	**Egg salad**
City Cafe	6	5
Sandwich City	3	12
Express Sandwiches	7	17
Sam's Sandwich Shop	1	6
Kelly's Subs	3	4

Question: As part of a project for health class, Cara surveyed local delis about the kinds of sandwiches sold. Which shop sold fewer sandwiches, Sandwich City or Express Sandwiches?
Options: (A) Sandwich City (B) Express Sandwiches
Answer: **(A) Sandwich City**
Solution:
Add the numbers in the Sandwich City row. Then, add the numbers in the Express Sandwiches row.
Sandwich City: 3 + 12 = 15. Express Sandwiches: 7 + 17 = 24.
15 is less than 24. **Sandwich City** sold fewer sandwiches.

Figure 1: A *free-text* problem example and a *multi-choice* problem example on the TabMWP dataset.

learning can be highly unstable across different selections of in-context examples (Zhao et al., 2021; Liu et al., 2022a; Lu et al., 2022b). It could be worse on TabMWP since its problems are distributed across multiple question types and diverse table layouts.

To alleviate this challenge, we further propose a novel approach that can learn to select in-context examples from a small amount of training data via policy gradient, termed PromptPG. As illustrated in Figure 2, an agent learns to find optimal in-context examples from a candidate pool, with the goal of maximizing the prediction rewards on given training examples when interacting with the GPT-3 environment. The policy network is built on top of the language model BERT (Devlin et al., 2018) with fixed parameters, followed by a one-layer linear neural network with learnable parameters. The learnable parameters are updated following the policy gradient strategy. Unlike random selection, brute-force search, or retrieval-based selection, PromptPG learns to construct the prompt dynamically given the candidate pool when interacting with the GPT-3 API.

Our contributions are as follows: (a) We present a new large-scale dataset, TabMWP, the first dataset for math word problems with tabular context; (b) We propose a novel approach, PromptPG, which learns to select in-context examples via policy gradient for few-shot GPT-3. To the best of our knowledge, it is the first work that applies reinforcement learning to select in-context examples; (c) Experimental results show that PromptPG achieves an improvement of up to 5.31% on TabMWP over existing methods, with reduced selection instability compared to random selection.

2 The TabMWP Dataset

Task Formulation. A tabular math word problem p is represented as a pair (t, q), where t is a table context and q is a question. The table t could be represented in a visual format as an image, semi-structured text, or a structured database. In this work, we focus on the semi-structured format as the table context for simplicity. Depending on the question and answer types, the question q may be accompanied by multiple-choice options $c = \{c_1, c_2, \dots, c_n\}$ or a unit u. Given a semi-structured tabular context t and an unstructured question text q, the task is to generate the answer a, which is either numerical only text for a *free-text* question, or a text span from given options for a *multiple-choice* question.

Key Statistics. We construct the TabMWP dataset that consists of 38,431 tabular math word problems. The dataset is partitioned with 6:2:2 into the training, development, and test splits, corresponding to 23,059, 7,686, and 7,686 problems. One distinct characteristic

Statistic	**Number**
Total questions	38,431
* *free-text* questions	28,719
* *multi-choice* questions	9,712
# of different questions	28,876
# of different answers	6,153
# of different solutions	35,442
# of different tables	37,644
# of tables with a title	23,259
# of table cells (Average/Max)	12.9 / 54
# of table rows (Average/Max)	5.9 / 11
# of table columns (Average/Max)	2.2 / 6
Question length (Average/Max)	22.1 / 92
Answer length (Average/Max)	1.1 / 27
Solution length (Average/Max)	49.5 / 350

Table 1: Key statistics for TabMWP.

of TABMWP is that each problem is accompanied by a tabular context, without which the problem would be unsolvable. Main statistics in Table 1 suggest that questions and tables in TABMWP distribute diversely across semantics and layouts. Details of data collection and more analysis are shown in Appendix B.

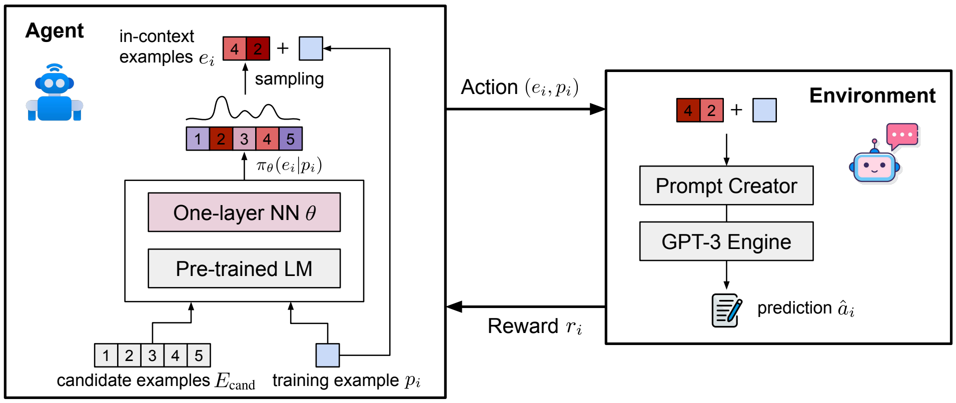

Figure 2: PROMPTPG learns to select good in-context examples for GPT-3 via policy gradient.

3 Our Method

Our model PROMPTPG is built on top of the few-shot GPT-3 model. A few in-context examples, along with the test example p_i, are provided to GPT-3 for the answer prediction. The examples can be randomly selected from the training set. Recent research, however, has shown that few-shot GPT-3 can be highly unstable across different selections of in-context examples (Zhao et al., 2021; Liu et al., 2022a; Lu et al., 2022b). To alleviate this issue, we aim to propose a novel approach that can learn to select performing in-context examples using a policy gradient strategy.

Formally, given a TABMWP problem p_i, we want the agent to find K in-context examples $e_i = \{e_i^1, e_i^2, ..., e_i^K\}$ from a candidate pool E_{cand}, and generate the answer $\hat{a}_i$, maximizing a reward $r_i = R(\hat{a}_i|p_i)$. The in-context examples are selected according to a policy

$$e_i^k \sim \pi_\theta(e_i|p_i),\ e_i^k \in E_{\text{cand}}, e_i^k \text{ are independent for } k = \{1, 2, ..., K\}, \tag{1}$$

where θ are the policy's parameters. The answer is generated through: $\hat{a}_i = \text{GPT-3}(e_i, p_i)$ using the selected examples and the given problem as the input prompt. The reward is then computed by evaluating the generated answer $\hat{a}_i$ with respect to the ground truth answer a_i:

$$r_i = R(\hat{a}_i|p_i) = \textsc{Eval}(\hat{a}_i, a_i),\ r_i \in \{-1, 1\}. \tag{2}$$

The function $\textsc{Eval}()$ returns a reward of 1 if the generated answer aligned with the label and -1 otherwise. Our goal is to maximize the expected reward of the generated answer under the policy $\mathbb{E}_{e_i \sim \pi_\theta(e_i|p_i)}[R(\text{GPT-3}(e_i, p_i))]$. We optimize the reward with respect to the parameters of the policy network using the Policy Gradient method (Sutton et al., 1998). The expected reward cannot be computed in closed form, so we compute an unbiased estimation with Monte Carlo Sampling,

$$\mathbb{E}_{e_i \sim \pi_\theta(e_i|p_i)}\left[R(\text{GPT-3}(e_i, p_i))\right] \approx \frac{1}{N}\sum_{i=1}^{N} R(\text{GPT-3}(e_i, p_i)),\ e_i \sim \pi_\theta(e_i|p_i), \tag{3}$$

where N is the size of each batch yielded from our training problem set P_{train}. In this work, we experiment using the REINFORCE policy gradient algorithm (Williams, 1992).

The policy is calculated as $\pi_\theta(e_i|p_i) = \frac{\exp\left[\mathbf{h}(e_i)\cdot\mathbf{h}(p_i)\right]}{\sum_{e_i' \in E_{\text{cand}}} \exp\left[\mathbf{h}(e_i')\cdot\mathbf{h}(p_i)\right]}$, where $\mathbf{h}()$ is the representation for the problem from BERT (Devlin et al., 2018) followed by an added linear layer. We add a small linear layer on top of the BERT final pooling layer. During training, the parameters of BERT are fixed and only the appended linear layer is updated. PROMPTPG is summarized in Algorithm 1 in Appendix.

Method	Training Data	Selection Strategy	Question Types		Answer Types					Grades		Avg.
			FREE	MC	INT	DEC	EXTR	BOOL	OTH	1-6	7-8	
Heuristic Baselines												
Heuristic guess	-	-	6.71	39.81	8.37	0.26	30.80	51.22	26.67	17.55	12.27	15.29
Human performance	-	-	84.61	93.32	84.95	83.29	97.18	88.69	96.20	94.27	81.28	90.22
pre-trained Baselines												
UnifiedQA$_{\text{BASE}}$	-	-	4.60	43.02	5.28	1.97	37.08	50.11	38.10	17.14	11.11	14.56
UnifiedQA$_{\text{LARGE}}$	-	-	4.48	48.80	5.19	1.72	48.33	50.33	40.00	19.78	10.87	15.96
TAPEX$_{\text{BASE}}$	-	-	7.32	39.76	8.68	2.06	35.06	47.11	20.95	18.67	11.81	15.73
TAPEX$_{\text{LARGE}}$	-	-	8.80	46.59	10.62	1.72	46.91	48.11	30.48	22.65	13.18	18.59
fine-tuned Baselines												
UnifiedQA$_{\text{BASE}}$	7,686	-	34.02	70.68	40.74	7.90	84.09	55.67	73.33	53.31	30.46	43.52
UnifiedQA$_{\text{LARGE}}$	7,686	-	48.67	82.18	55.97	20.26	94.63	68.89	79.05	65.92	45.92	57.35
TAPEX$_{\text{BASE}}$	7,686	-	39.59	73.09	46.85	11.33	84.19	61.33	69.52	56.70	37.02	48.27
TAPEX$_{\text{LARGE}}$	7,686	-	51.00	80.02	59.92	16.31	**95.34**	64.00	73.33	67.11	47.07	58.52
Prompting Baselines w/ GPT-3												
Zero-shot	-	-	53.57	66.67	55.55	45.84	78.22	55.44	54.29	63.37	48.41	56.96
Zero-shot-CoT	-	-	54.36	66.92	55.82	48.67	78.82	55.67	51.43	63.62	49.59	57.61
Few-shot (2-shot)	2	Random	54.69	64.11	58.36	40.40	75.95	52.41	53.02	63.10	49.16	57.13
Few-shot-CoT (2-shot)	2	Random	60.76	69.09	60.04	63.58	76.49	61.19	**67.30**	68.62	55.31	62.92
PROMPTPG *w/ GPT-3 (Ours)*												
Few-shot-CoT (2-shot)	160+20	Dynamic	**66.17**	**74.11**	**64.12**	**74.16**	76.19	**72.81**	65.71	**71.20**	**64.27**	**68.23**$_{5.31\uparrow}$

Table 2: Evaluation results of various baselines and our method on TABMWP. Training Data: number of used training data; Selection Strategy: strategy of selecting in-context examples for few-shot GPT-3; FREE: *free-text* questions; MC: *multi-choice* questions; INT: integer answers; DEC: decimal answers; EXTR: extractive text answers; BOOL: Boolean text answers; OTH: other text answers.

4 Experiments

Baselines and evaluation metric. We evaluate our TABMWP dataset on various recent models, including UnifiedQA (Khashabi et al., 2020) and TAPEX (Liu et al., 2022b) in both pre-trained and fine-tuned settings. We implement the zero-shot GPT-3 model, the few-shot GPT-3 model, and their chain-of-thought (CoT) reasoning variants (Wei et al., 2022). We also study the heuristic guess baseline and human performance to analyze the lower and upper bounds on TABMWP, respectively. The answer part is extracted from the GPT-3 generation using manually designed regular expressions. We determine if the generated answer is correct given the ground truth and use accuracy as the metric. For *free-text* problems where the answer is set as a number, we normalize the prediction and the label to decimal numbers with two-digit precision and check if their values are equivalent. For *multi-choice* problems, we choose the most similar one from options following Khashabi et al. (2020).

Results. Table 2 demonstrates the results of different methods on TABMWP. Benefiting from pre-training on the tabular corpus, TAPEX performs better on average than UnifiedQA. Increasing the model size can improve the prediction accuracy for both UnifiedQA and TAPEX. Fine-tuned on TABMWP, the baseline models can significantly improve the prediction performance. Without any example provided to GPT-3, zero-shot GPT-3 achieves a comparable accuracy as the best fine-tuned baselines UnifiedQA$_{\text{LARGE}}$ and TAPEX$_{\text{LARGE}}$. Provided with two randomly sampled in-context examples, few-shot GPT-3 gets an improvement of 0.17%. Generating the multi-step solution before the answer, the few-shot-CoT GPT-3 model reports the best performance among all of these baseline models, with an accuracy of 62.92%. Unlike few-shot-CoT GPT-3 randomly selecting the in-context examples, our proposed PROMPTPG learns to select performing examples with the help of policy gradient. PROMPTPG establishes a state-of-the-art performance on the TABMWP dataset: it surpasses the best baseline few-shot-CoT GPT-3 by 5.31% on average. PROMPTPG shows its consistent advantages on two question types, two grade groups, and most of the answer types.

Case Study. We conduct the case study in Appendix D.5. We visualize the two in-context examples selected by strategies of our PROMPTPG, nearest neighbor search, and random selection, in Figure 5, 6, and 7, respectively. The nearest neighbor search strategy selects the "superficially" similar examples to the test example. Instead, PROMPTPG tends to select examples that have multiple reasoning steps in the solution and similar abilities in mathematical reasoning, which results in higher prediction accuracy. Successful examples in Figure 8 - 12 show that PROMPTPG is able to generate reasonable reasoning steps to predict correct answers for a wide range of TABMWP problems. Failure examples in Figure 13 - 18 suggest that PROMPTPG has limitations when solving problems provided with complex tabular contexts or requiring a high-level ability of mathematical reasoning. For more implementation details and results analysis, please refer to Appendix D.3.

5 Conclusion

In this paper, we propose TABMWP, the first large-scale dataset for math word problems in tabular contexts. We propose a novel approach, PROMPTPG, for few-shot GPT-3, which utilizes policy gradient to learn to select good in-context examples from the training data to construct the prompt for the test example. Experimental results show that PROMPTPG outperforms existing strong baselines by 5.31% and reduces the accuracy volatility compared to random selection.

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Supplementary Materials for Learn to Select Good Examples with Reinforcement Learning for Semi-structured Mathematical Reasoning

A Related Work

A.1 Math Word Problems

The task of solving Math Word Problems (MWPs) is to predict the answer given a natural language description of a math problem. There have been great efforts in developing datasets for MWPs, including Dolphin18K (Huang et al., 2016), DRAW-1K (Upadhyay & Chang, 2017), Math23K (Wang et al., 2017), MathQA (Amini et al., 2019), ASDiv (Miao et al., 2020), and SVAMP (Patel et al., 2021). However, these datasets only involve the textual modality, and most are limited to a small data scale. Some recent datasets like DVQA (Kafle et al., 2018), Geometry3K (Lu et al., 2021a) and IconQA (Lu et al., 2021b) introduce math problems with diagrams as the visual context, where the system needs to perform mathematical reasoning over multi-modal information. To the best of our knowledge, our dataset TABMWP is the first dataset that requires mathematical reasoning over heterogeneous information from both the textual question and the tabular context. To solve MWPs, one popular line of previous methods is to generate the intermediate expressions and execute them to get the final answers (Huang et al., 2017; Roy & Roth, 2017; Amini et al., 2019). Inspired by the recent progress achieved by GPT-3 in solving MWPs (Wei et al., 2022; Wang et al., 2022; Kojima et al., 2022), we evaluate TABMWP using GPT-3 models in zero-shot and few-shot learning manners.

A.2 Table QA Datasets

Table Question Answering (Table QA) refers to the task of answering questions about tabular data. Numerous datasets have been developed for Table QA. For example, TabMCQ (Jauhar et al., 2016) is an early dataset collected from grade exams. Datasets like WTQ (Pasupat & Liang, 2015), WikiSQL (Zhong et al., 2017), and SQA (Iyyer et al., 2017) contain semi-structured tables from Wikipedia, while Spider (Yu et al., 2018) collects structured tables sourced from databases. Recent work aims at introducing datasets that require multi-hop reasoning between the textual and tabular data: HybridQA (Chen et al., 2020b), OTTQA (Chen et al., 2020a), MultiModalQA (Talmor et al., 2020), AIT-QA (Katsis et al., 2021), and FeTaQA (Nan et al., 2022). Datasets most related to our TABMWP dataset are FinQA (Chen et al., 2021), TAT-QA (Zhu et al., 2021), and MultiHiertt (Zhao et al., 2022) because they need numerical reasoning on financial reports with tabular data. Note that 77.6% of questions in TAT-QA can be solvable without mathematical reasoning and 50.0% of questions in FinQA are not table-must to be answered. In contrast, our proposed TABMWP collects questions where both mathematical reasoning and tabular context are necessary.

A.3 Prompt Learning for Language Models

Large pre-trained language models, such as GPT-3 (Brown et al., 2020), have shown their remarkable ability of few-shot learning on a wide range of downstream tasks (Houlsby et al., 2019; Brown et al., 2020; Lu et al., 2022a). Given a few in-context examples as demonstrations, GPT-3 can generalize to unseen test examples without parameter updating. For example, Wei et al. (2022) randomly select different in-context examples from the training set and formulate their corresponding prompt with a test sample. However, recent studies show that few-shot GPT-3 highly depends on the selection of in-context examples and could be unstable, varying from the near chance to near state-of-the-art performance (Zhao et al., 2021; Liu et al., 2022a). To mitigate the volatility of selecting in-context examples, Lu et al. (2022b) propose retrieving relevant examples that are semantically similar to the test sample. Other possible strategies could be using brute-force permutation search or relying on manually designed heuristics like choosing the most complex examples. Inspired by reinforcement learning's ability to search for an optimal action policy, we propose applying the policy gradient strategy (Sutton et al., 1998) to learn to select in-context examples more efficiently and stably without designing human-designed heuristics.

B Dataset

B.1 Dataset Construction

Data collection. The raw problems are collected from an online learning website, IXL[2], which hosts a large number of high-quality math problems curated by educational experts. Only math word problems that are accompanied by a tabular context and a detailed solution are collected. We develop a script to extract the tabular context, the question, options that apply, the correct answer, and the solution for each problem. These elements can be precisely identified using HTML tags. For each table, we take a screenshot and store its raw text.

Data preprocessing. To make TABMWP compatible with various baselines, we represent the tabular context as three formats: an image, *semi-structured* text, and a *structured* spreadsheet. The semi-structured format is created by converting the raw table text into a flattened token sequence, with each row separated by a newline character '`\n`' and each column separated by '|'. The semi-structured text is further transformed to the structured format, which can be easily retrieved and executed by SQL-based methods (Liu et al., 2022b) using packages like `pandas`. For clarity, the table title is separated from the raw table. Examples of three formats are shown in Table 4.

Image format

Field day schedule		
Event	**Begin**	**End**
water balloon toss	11:30 A.M.	11:50 A.M.
obstacle course	12:05 P.M.	12:25 P.M.
parachute ball toss	12:30 P.M.	1:30 P.M.
jump rope race	1:40 P.M.	2:05 P.M.
balloon stomp	2:15 P.M.	2:35 P.M.
relay race	2:50 P.M.	3:40 P.M.
hula hoop contest	3:55 P.M.	4:30 P.M.
potato sack race	4:40 P.M.	5:15 P.M.

Semi-structured format

Table title: Field day schedule
Table text:
Event | Begin | End
water balloon toss | 11:30 A.M. | 11:50 A.M.
obstacle course | 12:05 P.M. | 12:25 P.M.
parachute ball toss | 12:30 P.M. | 1:30 P.M.
jump rope race | 1:40 P.M. | 2:05 P.M.
balloon stomp | 2:15 P.M. | 2:35 P.M.
relay race | 2:50 P.M. | 3:40 P.M.
hula hoop contest | 3:55 P.M. | 4:30 P.M.

Structured format

Table title: Field day schedule

	Event	Begin	End
0	water balloon toss	11:30 A.M.	11:50 A.M.
1	obstacle course	12:05 P.M.	12:25 P.M.
2	parachute ball toss	12:30 P.M.	1:30 P.M.
3	jump rope race	1:40 P.M.	2:05 P.M.
4	balloon stomp	2:15 P.M.	2:35 P.M.
5	relay race	2:50 P.M.	3:40 P.M.
6	hula hoop contest	3:55 P.M.	4:30 P.M.
7	potato sack race	4:40 P.M.	5:15 P.M.

Table 3: Three different formats for the tables in the TABMWP dataset.

For better quantitative evaluation, we formalize the TABMWP problems as two question types: (a) *free-text* questions, where the answer is numerical text only and the unit text is separately extracted; and (b) *multi-choice* questions, the answer of which is the text span from choice options. Descriptions of these question and answer types are shown in Table 4. The order of choice options is shuffled to alleviate distribution bias. Redundant information in solutions is removed, and some solutions are manually rewritten to be more human-readable. Finally, problems with the same table, question, and answer text are regarded as redundant and thus removed.

Question types	Answer types (%)	Descriptions
Free-text	Integer (59.50%)	The answer is an integer number, e.g., "40", "1,207", "-3".
	Decimal (15.23%)	The answer is a decimal or a fraction number, e.g., "192.80", "68/217".
Multi-choice	Extractive (13.01%)	The answer could be extracted from the table context.
	Boolean (10.97%)	The answer is Boolean, e.g., "yes"/"no", "true"/"false", "linear"/"nonlear".
	Other (1.29%)	The answer belongs to other text types, e.g., a statement.

Table 4: Format diversity of questions and answers in TABMWP.

Quality control. The goal of constructing TABMWP is to collect math word problems that necessitate multi-hop mathematical reasoning between the question and the tabular context. Therefore, we ask human experts to filter problems that can be solved either without the context of the table or by looking up table cells without numerical reasoning. To further ensure data quality, we ask human experts to perform a final review to re-check the dataset and manually revise incorrect annotations.

[2] `https://www.ixl.com/math`

B.2 Comparison to existing datasets

Comparison to existing datasets. As shown in Table 5, TABMWP differs from related datasets in various aspects: (1) TABMWP is the first dataset to study math word problems over tabular context on open domains and is the largest in terms of data size; (2) Problems in TABMWP are annotated with the tabular context, unlike previous MWP datasets in the first segment; (3) Different from Table QA datasets like FinQA, TAT-QA, and MultiHiertt, a lack of either mathematical reasoning or the tabular context renders the problems in TABMWP unanswerable; (4) There are two question types in TABMWP, and the answer could be a text span, an integer number, or a decimal number; (5) Each problem is annotated with natural language solutions to reveal multi-hop reasoning steps.

Dataset	Size	#Table	Need Math?	Need Table?	Table Type		Question Type		Answer Type			Solution Type
					Domain	Format	Free-text	MC	Text	Integer	Decimal	
Dolphin18K (2016)	831	✗	✓	✗	✗	✗	✓	✗	✗	✓	✓	formula
DRAW-1K (2017)	1,000	✗	✓	✗	✗	✗	✓	✗	✗	✓	✓	formula
Math23K (2017)	23,162	✗	✓	✗	✗	✗	✓	✗	✗	✓	✓	formula
MathQA (2019)	37,297	✗	✓	✗	✗	✗	✗	✓	✗	✓	✓	formula
ASDiv (2020)	2,305	✗	✓	✗	✗	✗	✓	✗	✓	✓	✓	formula
SVAMP (2021)	1,000	✗	✓	✗	✗	✗	✓	✗	✗	✓	✗	formula
GSM8K (2021)	8,792	✗	✓	✗	✗	✗	✓	✗	✗	✓	✗	text
IconQA (2021b)	107,439	✗	✓	✗	✗	✗	✓	✓	✓	✓	✗	✗
FinQA (2021)	8,281	2,766	✓	76.6%	finance	text	✓	✗	✗	✓	✓	program
TAT-QA (2021)	16,552	2,747	50.0%	✓	finance	text	✓	✗	✗	✓	✓	✗
MultiHiertt (2022)	10,440	9,843	✓	89.8%	finance	text	✓	✗	✗	✓	✓	✗
TABMWP (ours)	**38,431**	**37,644**	✓	✓	**open**	**text***	✓	✓	✓	✓	✓	**text**

Table 5: A comparison of MWP and Table QA datasets that require numerical reasoning. *text**: each table in TABMWP is accompanied by an image format.

B.3 Human study

To examine how humans perform on our TABMWP dataset, we released the human evaluation task on Amazon Mechanical Turk (AMT) to the test split. We designed two sub-tasks for the human study: answering the *free-text* questions and answering the *multi-choice* questions. The user interfaces for the two sub-tasks are shown in Figure 3. Each human intelligence task (HIT) contains 5 exam questions and 15 test questions. A worker should have a HIT Approval Rate of 98% or higher and be approved with 5,000 or more HITs. The worker is provided with detailed instructions at the beginning and needs to pass at least 3 *free-text* exam questions or 4 *multi-choice* exam questions to be qualified for the human study. Each HIT is assigned to two different workers. We assign a reward of $0.80 and $0.60 for one HIT of *free-text* and *multi-choice* sub-tasks, respectively.

C The PROMPTPG Algorithm

The pipeline of PROMPTPG to learn to select in-context examples is summarized in Algorithm 1.

D Experimental Details and More Results

D.1 Baselines

Heuristics guess. To investigate the lower bound of the accuracy on TABMWP, we design simple heuristics to guess answers for each question type. For *multi-choice* questions, we randomly select one from the given options with even probabilities. For *free-text* questions on TABMWP, the answers could only be integral or decimal numbers. Intuitively, we take advantage of regular expressions to extract all the numbers from the tabular context and the question text as candidates, and then randomly choose one number as the prediction.

UnifiedQA baselines. UnifiedQA (Khashabi et al., 2020) is a T5-based (Raffel et al., 2020) QA system that was pre-trained on 8 seed QA datasets of multiple formats but with a unified text-to-text paradigm. We load the pre-trained checkpoint as the pre-trained baseline and train it on TABMWP as the fine-tuned baseline. Three different parameter sizes are compared: SMALL (60M), BASE (220M), and LARGE (770M).

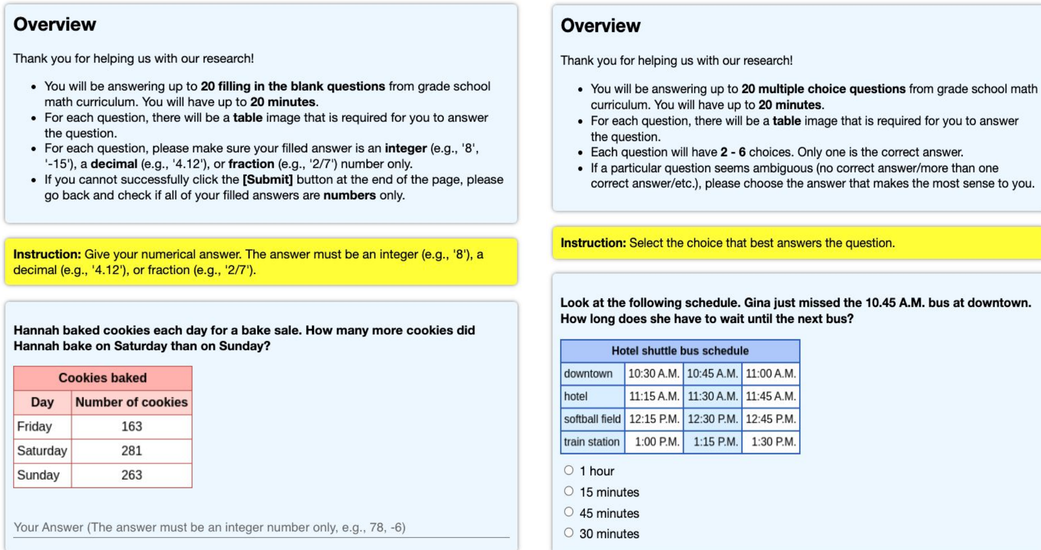


Figure 3: User interfaces of human study for *free-text* and *multi-choice* questions.

Algorithm 1 Dynamic Prompt Learning via Policy Gradient (PROMPTPG)

Input: Initial policy π_{θ_0}, training example set P_{train}, candidate example set E_{cand}, # of training epochs N
Output: Learned policy π_θ

function REINFORCE(π_{θ_0}, P_{train}, E_{cand}, N)
  Initialize policy network π with parameter θ_0
  for epoch = $1, 2, ..., N$ **do**
    for $P_{\text{batch}} \in P_{\text{train}}$ **do** ▷ get a batch from the training set
      $\mathcal{L}_{\text{batch}} \leftarrow 0$
      for $p_i \in P_{\text{batch}}$ **do**
        Sample $e_i^k \sim \pi_\theta(e_i|p_i), e_i^k \in E_{\text{cand}}, k = \{1, ..., K\}$ ▷ K is # of in-context examples
        $\hat{a}_i \leftarrow$ GPT-3$(e_i^1, ..., e_i^k, p_i)$ ▷ $\hat{a}_i$ is the GPT-3 generated answer
        $r_i \leftarrow$ EVAL$(\hat{a}_i, a_i), r_i \in \{-1, 1\}$ ▷ a_i is the ground truth answer of p_i
        $\mathcal{L}_{\text{batch}} \leftarrow \mathcal{L}_{\text{batch}} - r_i \cdot \ln \pi_\theta(e_i|p_i)$
      end for
      Optimize $\mathcal{L}_{\text{batch}}$ wrt. θ
    end for
  end for
  return π_θ
end function

TAPEX baselines. TAPEX (Liu et al., 2022b) is a BART-based (Lewis et al., 2020) language model pre-trained on structured tabular data to mimic the behavior of a SQL executor that can answer table-based questions. TAPEX shows state-of-the-art performance on four table-related datasets. We establish the pre-trained and fine-tuned baselines on top of TAPEX with two model sizes: BASE (140M) and LARGE (400M).

Zero-shot GPT-3 and zero-shot-CoT GPT-3. We establish the zero-shot baseline based on GPT-3 (Brown et al., 2020). The zero-shot setup follows the format of TQ(C)→A where the input is the concatenation of tokens of the tabular context (T), the question text (Q), and choice options (C) that apply while the output is to predict the answer (A). Following Kojima et al. (2022), we further build zero-shot-CoT GPT-3, which refers to the GPT-3 model with a chain-of-thought (CoT) prompt. Specifically, we add the prompt "*Let's think step by step*" at the end of the input to ask the model to generate the multi-step solution (S) to mimic the reasoning process as humans. Then the model takes the raw input and the newly generated solution to predict the final answer.

Few-shot GPT-3 and few-shot-CoT GPT-3. In the few-shot setting, we follow the standard prompting (Wei et al., 2022) where in-context examples are randomly selected from the training data as demonstrations for the text example. Similarly, the few-shot-CoT GPT-3 baseline takes the prompt template of TQ(C)$\rightarrow$SA to generate the solution before the final answer.

D.2 Implementation Details

Fine-tuned UnifiedQA and TAPEX baselines are trained on the train split and evaluated on the test split. We use the Adam optimizer (Kingma & Ba, 2014) with an initial learning rate of $5\mathrm{e}{-5}$. The training process takes 10 epochs with a batch size of 16. The maximum number of input tokens is set as 200 and the maximum output length is 100.

Experiments for two few-shot GPT-3 baselines and our PROMPTPG are repeated three times, and the average accuracy is reported in Table 2. Few-shot GPT-3 and few-shot-CoT GPT-3 randomly select two in-context examples from the training data to build the prompt. Our PROMPTPG is built on top of few-shot GPT-3 with a different selection strategy: (a) in the training stage, the agent learns to select two examples from 20 candidates and is evaluated on 160 training examples to calculate the reward; (b) in the test stage, the agent with an optimal policy chooses two examples from 20 candidates for each test example. The candidates are randomly selected from the training set.

In our proposed PROMPTPG, the embedding size of the added linear neural network is 768. To learn the policy network, we use the Adam optimizer with an initial learning rate of $1\mathrm{e}{-3}$. The maximum number of training epochs is 30, with a batch size of 20. The training process is stopped early if there is any NaN value in the loss for a batch of training data.

Our experiments for UnifiedQA baselines, TAPEX baselines, and our proposed PROMPTPG are conducted using PyTorch on two Nvidia RTX 3090 GPUs. For the GPT-3 engine, we use TEXT-DAVINCI-002, the most capable engine recommended by the official documentation. The temperature is set as 0 and the top probability is set as 1.0 to get the most deterministic prediction. The maximum number of tokens allowed for generating text is 512. Both the frequency penalty and the presence penalty are set as the default value, i.e., 0.

D.3 More Experimental Results

Heuristic guess and human performance. The accuracy of *multi-choice* questions by heuristic guess is 39.81%, which aligns with the fact that there are 2.88 options on average. The accuracy for *free-text* questions is considerably low since the inputs of TABMWP problems do not have direct clues for the answers. Humans outperform all benchmarks consistently across question types, answer types, and grade groups, with a 21.99% average accuracy advantage over our best performing PROMPTPG. This gap is to be filled by future research on semi-structured mathematical reasoning.

Problem types and difficulty. Among all the baselines, we find it is easier for models to answer *multi-choice* questions than *free-text* questions. Questions with the boolean (BOOL) and other (OTH) answer types tend to have lower accuracy scores than the extractive (EXTR) answer type, because the former ones need the abilities of fact verification and language understanding on diverse options, respectively. It is also not surprising for us to find that all the models perform worse on problems in grades 7-8 than in a lower-level group of 1-6.

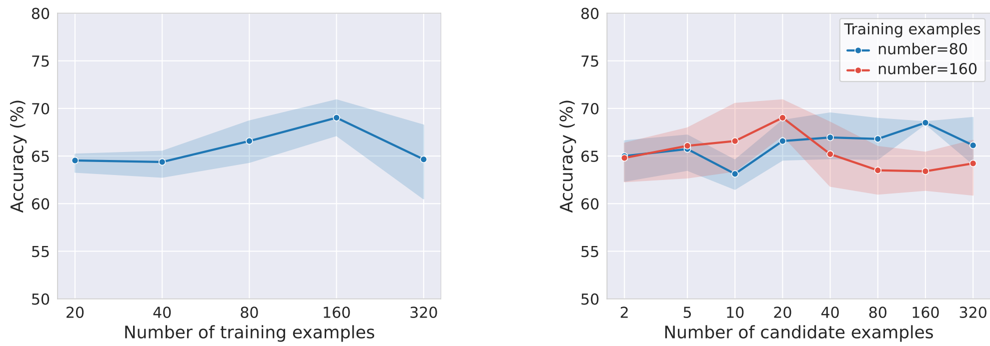
D.4 Ablation Study

Here, we will study how different factors have an effect on the performances of baselines and our method on TABMWP. Experiments are conducted on 1,000 development examples.

Blind study of the dataset. We evaluate the information gain of each component of the TABMWP problems by removing it from model inputs. To eliminate the impact and variance caused by example selection, the study is conducted using the zero-shot GPT-3 model. As shown in Table 6, there is a dramatic decline when either the tabular context (T) or the question text (Q) is missing from the inputs. For example, T$\rightarrow$A and Q$\rightarrow$A only attain an average accuracy of 6.10% and 7.00%, respectively, and their accuracies are near to zero on the *multi-choice* questions. Taking both tabular and textual data as inputs (TQ$\rightarrow$A), the model significantly beats the heuristic guess. With the complete input information (TQ(C)$\rightarrow$A), the full model achieves the best performance. The blind study shows that our TABMWP is robust and reliable in distribution, and all input components are indispensable parts that provide necessary information for answering the questions.

Model	Format	FREE	MC	INT	DEC	EXTR	BOOL	OTH	1-6	7-8	Avg.
Heuristic guess	TQ(C)→A	7.31	40.36	9.20	0.00	34.44	47.32	50.00	17.99	13.96	16.40
Zero-shot GPT-3	T→A	8.28	0.36	10.24	0.67	0.66	0.00	0.00	9.41	1.02	6.10
Zero-shot GPT-3	Q→A	9.24	1.09	10.94	2.68	1.32	0.89	0.00	10.23	2.03	7.00
Zero-shot GPT-3	T(C)→A	8.28	41.82	10.24	0.67	36.42	50.89	25.00	23.60	8.12	17.50
Zero-shot GPT-3	Q(C)→A	9.10	33.09	10.94	2.01	25.17	44.64	25.00	21.29	7.11	15.70
Zero-shot GPT-3	TQ→A	55.31	68.36	56.60	50.34	79.47	54.46	58.33	66.34	47.46	58.90
Zero-shot GPT-3 (full model)	TQ(C)→A	54.76	72.00	56.42	48.32	76.82	66.07	66.67	67.00	47.97	59.50

Table 6: Blind studies on TABMWP. T: tabular context; Q: question; C: choice options; A: answer.



(a) Accuracy w.r.t. different numbers of training examples, given 20 candidate examples.

(b) Accuracy w.r.t. different numbers of candidates, given 80 and 160 training examples.

Figure 4: Accuracy w.r.t. different numbers of training and candidate examples. Experiments are conducted on 1,000 development instances, and each setting is repeated with four random seeds.

Number of training examples. We study the effect of different numbers of training examples on our dynamic prompt learning in Figure 4 (a). With more training examples, the prediction accuracy first gradually increases to a peak of around 160 training examples. After that, the accuracy goes down with a growing variance. We reckon it is because the policy gradient algorithm can benefit from the scaling-up training data but fails to exploit more examples efficiently.

Number of candidate examples. In Figure 4 (b), we investigate how different numbers of candidate examples can affect policy learning performance. With the increasing candidate number, it is observed that the prediction accuracy will first go up and then go down after a threshold, given 80 or 160 training examples. It is probably because when the candidate pool is too small, the policy gradient algorithm has a limited action space to explore enough problem types. In contrast, too many candidates could make the algorithm hard to learn an optimal policy in a large search space.

Different selection strategies. In Table 7, we compare the proposed PROMPTPG with random selection and other heuristic-based example selection strategies for the few-shot-CoT GPT-3 model. Compared to random selection, selecting the same question or answer type of examples helps the model to take the task-relevant examples as the prompt, thus improving the accuracy and reducing the variance. Choosing the most complex examples does not boost the prediction performance consistently. The most semantically similar examples, as a kind of nearest neighbor search of the test example, help construct the performing and stable prompt for GPT-3. PROMPTPG shows its effectiveness in selecting optimal in-context examples over other strategies and largely reduces the instability caused by randomness.

Selection strategy	Acc. (%)
Same question type	66.2 ± 0.60
Same answer type	67.9 ± 0.38
Same grade level	67.9 ± 1.87
Most complex (# of table cells)	64.0 ± 0.42
Most complex (# of ques. words)	68.2 ± 0.26
Random selection	65.2 ± 4.01
Nearest neighbor	68.2 ± 0.29
PROMPTPG (Ours)	$\mathbf{70.9} \pm \mathbf{1.27}$

Table 7: Evaluation results w.r.t. different strategies for selecting in-context examples.

D.5 Case study examples

▷ *In-context example 1 (ID: 28463)*

Table:
Option | Change in phone price
Add an upgrade | $60
Buy a used phone | -$75

Option	Change in phone price
Add an upgrade	$60
Buy a used phone	-$75

Question: Luna is looking at the price of new cell phones online. Her favorite company, OrangeTech, has a special this weekend. Luna can add an upgrade to a phone for an additional cost, or she can buy a used phone to get a discount. The change in price for each option is shown in the table. Which option results in a greater change in price?
Options: (A) adding an upgrade (B) buying a used phone
Answer:
(Step 1) To find the option that results in a greater change in price, use absolute value. Absolute value tells you how much the price changes.
(Step 2) Add an upgrade: |$60| = $60
(Step 3) Buy a used phone: |-$75| = $75
(Step 4) Buying a used phone results in a greater change in price. It reduces the price by $75. The answer is buying a used phone.

▷ *In-context example 2 (ID: 13974)*

Table:
heart-shaped beads | $3/kilogram
rectangular beads | $2/kilogram
spherical beads | $2/kilogram
oval beads | $2/kilogram

heart-shaped beads	$3/kilogram
rectangular beads	$2/kilogram
spherical beads	$2/kilogram
oval beads	$2/kilogram

Question: Rebecca bought 2.5 kilograms of oval beads. How much did she spend? (Unit: $)
Answer:
(Step 1) Find the cost of the oval beads. Multiply the price per kilogram by the number of kilograms.
(Step 2) $2 × 2.5 = $5
(Step 3) She spent $5. The answer is 5.

▷ *Test example (ID: 17417)*

Table:
[TITLE]: Birthday party
Activity | Parents | Children
Singing | 14 | 20
Eating cake | 5 | 10
Jumping rope | 16 | 20
Swimming | 16 | 19
Playing tag | 4 | 9

Birthday party		
Activity	**Parents**	**Children**
Singing	14	20
Eating cake	5	10
Jumping rope	16	20
Swimming	16	19
Playing tag	4	9

Question: At Josie's birthday party, children and parents celebrated by participating in some activities. How many more children than parents are playing tag? (Unit: children)
Answer:
(Step 1) To find the difference between the number of children and parents playing tag, subtract the number of parents from the number of children.
(Step 2) 9 - 4 = 5
(Step 3) There are 5 more children than parents playing tag. The answer is 5.
Output: 5

Figure 5: Two in-context examples selected by PROMPTPG, the prompt, and the **correct** prediction. The selected examples require similar abilities of mathematical reasoning to the test example.

▷ *In-context example 1 (ID: 18429)*

Table:
[TITLE]: Children's weights (lbs)
Stem | Leaf
1 | 7
2 | 4
3 |
4 |
5 | 2, 2, 8
6 | 6
7 | 1, 3
8 | 7, 8
9 | 0

Children's weights (lbs)	
Stem	**Leaf**
1	7
2	4
3	
4	
5	2, 2, 8
6	6
7	1, 3
8	7, 8
9	0

Question: Dr. Thornton, a pediatrician, weighed all the children who recently visited her office. How many children weighed exactly 52 pounds? (Unit: children)
Answer:
(Step 1) For the number 52, the stem is 5, and the leaf is 2. Find the row where the stem is 5. In that row, count all the leaves equal to 2.
(Step 2) You counted 2 leaves, which are blue in the stem-and-leaf plot above.
(Step 3) 2 children weighed exactly 52 pounds. The answer is 2.

▷ *In-context example 2 (ID: 18129)*

Table:
whole wheat pancake mix | $1/lb
buttermilk pancake mix | $2/lb
chocolate chip pancake mix | $2/lb
blueberry pancake mix | $1/lb
plain pancake mix | $2/lb
raspberry pancake mix | $2/lb

whole wheat pancake mix	$1 per lb
buttermilk pancake mix	$2 per lb
chocolate chip pancake mix	$2 per lb
blueberry pancake mix	$1 per lb
plain pancake mix	$2 per lb
raspberry pancake mix	$2 per lb

Question: If Vince buys 3.4 pounds of buttermilk pancake mix, how much will he spend? (Unit: $)
Answer:
(Step 1) Find the cost of the buttermilk pancake mix. Multiply the price per pound by the number of pounds.
(Step 2) $2 × 3.4 = $6.80
(Step 3) He will spend $6.80. The answer is 6.80.

▷ *Test example (ID: 17417)*

Table:
[TITLE]: Birthday party
Activity | Parents | Children
Singing | 14 | 20
Eating cake | 5 | 10
Jumping rope | 16 | 20
Swimming | 16 | 19
Playing tag | 4 | 9

Birthday party		
Activity	**Parents**	**Children**
Singing	14	20
Eating cake	5	10
Jumping rope	16	20
Swimming	16	19
Playing tag	4	9

Question: At Josie's birthday party, children and parents celebrated by participating in some activities. How many more children than parents are playing tag? (Unit: children)
Answer:
(Step 1) For the activity "Playing tag", the stem is 4, and the leaf is 9. Find the row where the stem is 4. In that row, count all the leaves equal to 9.
(Step 2) You counted 9 leaves, which are blue in the stem-and-leaf plot above. 9 children are playing tag. The answer is 9.
Output: 9
Ground truth: 5

Figure 6: Two in-context examples selected by *nearest neighbor search*, the prompt, and the final **wrong** prediction. The selected examples are only semantically similar to the test example.

▷ *In-context example 1 (ID: 13033)*

Table:
[TITLE]: Watermelons harvested
Day | Number of watermelons
Wednesday | 59
Thursday | 51
Friday | 53
Saturday | 52

Watermelons harvested	
Day	**Number of watermelons**
Wednesday	59
Thursday	51
Friday	53
Saturday	52

Question: A farmer recalled how many watermelons were harvested in the past 4 days. On which day were the most watermelons harvested?
Options: (A) Wednesday (B) Thursday (C) Friday (D) Saturday
Answer:
(Step 1) Find the greatest number in the table. Remember to compare the numbers starting with the highest place value. The greatest number is 59.
(Step 2) Now find the corresponding day. Wednesday corresponds to 59. The answer is Wednesday.

▷ *In-context example 2 (ID: 32386)*

Table:
[TITLE]: Basketball hoops
Park | Number of basketball hoops
Heron Park | 2
Kelly Park | 7
Westfield Park | 4
Pinehurst Park | 4
Linden Park | 3
Mooreland Park | 7
Crestview Park | 2
Riverfront Park | 4

Basketball hoops	
Park	**Number of basketball hoops**
Heron Park	2
Kelly Park	7
Westfield Park	4
Pinehurst Park	4
Linden Park	3
Mooreland Park	7
Crestview Park	2
Riverfront Park	4

Question: The parks department compared how many basketball hoops there are at each park. What is the mode of the numbers?
Answer:
(Step 1) Read the numbers from the table.
(Step 2) First, arrange the numbers from least to greatest: 2, 2, 3, 4, 4, 4, 7, 7.
(Step 3) Now count how many times each number appears.
(Step 4) 2 appears 2 times. 3 appears 1 time. 4 appears 3 times. 7 appears 2 times.
(Step 5) The number that appears most often is 4.
(Step 6) The mode is 4. The answer is 4.

▷ *Test example (ID: 17417)*

Table:
[TITLE]: Birthday party
Activity | Parents | Children
Singing | 14 | 20
Eating cake | 5 | 10
Jumping rope | 16 | 20
Swimming | 16 | 19
Playing tag | 4 | 9

Birthday party		
Activity	**Parents**	**Children**
Singing	14	20
Eating cake	5	10
Jumping rope	16	20
Swimming	16	19
Playing tag	4	9

Question: At Josie's birthday party, children and parents celebrated by participating in some activities. How many more children than parents are playing tag? (Unit: children)
Answer:
(Step 1) Find the difference between the number of children and the number of parents for each activity.
(Step 2) For singing, the difference is 6.
(Step 3) For eating cake, the difference is 5.
(Step 4) For jumping rope, the difference is 4.
(Step 5) For swimming, the difference is 3.
(Step 6) For playing tag, the difference is 5.
(Step 7) The greatest difference is 6. The answer is 6.
Output: 6
Ground truth: 5

Figure 7: Two *randomly* selected in-context examples, the prompt, and the final prediction. The selected examples have limited relevance to the test example, resulting in the **wrong** output.

Table:
[TITLE] Math teachers
High school | Number of math teachers
Central High | 9
Hillview High | 10
Westside High | 4
Moore High | 2
River High | 6
Northside High | 4
Lincoln High | 8
Thompson High | 5

Math teachers	
High school	**Number of math teachers**
Central High	9
Hillview High	10
Westside High	4
Moore High	2
River High	6
Northside High	4
Lincoln High	8
Thompson High	5

Question: The school district compared how many math teachers each high school has. What is the mean of the numbers?
Answer:
(Step 1) The mean is the average of the numbers.
(Step 2) To find the mean, add all the numbers and divide by the number of items.
(Step 3) 9 + 10 + 4 + 2 + 6 + 4 + 8 + 5 = 48
(Step 4) There are 8 items, so the mean is 48 ÷ 8 = 6.
(Step 5) The mean of the numbers is 6.
Output: 6

Figure 8: The **correct** prediction from our PROMPTPG for a *free-text* question example. This example requires taking the mean of eight numbers from the table via addition and division.

Table:
topaz | $18.55 per lb
amethyst | $19.88 per lb
tiger's eye | $10.29 per lb
fool's gold | $16.00 per lb
quartz | $14.63 per lb
calcite | $15.39 per lb
granite | $19.23 per lb

topaz	$18.55 per lb
amethyst	$19.88 per lb
tiger's eye	$10.29 per lb
fool's gold	$16.00 per lb
quartz	$14.63 per lb
calcite	$15.39 per lb
granite	$19.23 per lb

Question: Austin went to the store and bought 3 pounds of fool's gold, 1 pound of topaz, and 2 pounds of tiger's eye. How much did he spend? (unit: $)
Answer:
(Step 1) To find the total amount Austin spent, multiply the price per pound of each type of stone by the number of pounds of that stone that he bought, and then add all of the products together.
(Step 2) 3 pounds of fool's gold: $16.00 per pound × 3 pounds = $48.00
(Step 3) 1 pound of topaz: $18.55 per pound × 1 pound = $18.55
(Step 4) 2 pounds of tiger's eye: $10.29 per pound × 2 pounds = $20.58
(Step 5) $48.00 + $18.55 + $20.58 = $87.13
(Step 6) Austin spent $87.13.
Output: 87.13

Figure 9: The **correct** prediction from our PROMPTPG for a *free-text* question example. This example requires looking up three items and their corresponding prices in the table, calculating their costs, and finally summing them up to get the final answer.

Table:
Employee | Pay period |
Dhruba Khanal | December 9-15 |
Total earnings | | $620.00
Federal income tax | $71.14 |
State income tax | $48.90 |
Other taxes | $47.00 |
Total taxes | | ?
Pay after taxes | | ?

Employee	Pay period	
Dhruba Khanal	December 9-15	
Total earnings		$620.00
Federal income tax	$71.14	
State income tax	$48.90	
Other taxes	$47.00	
Total taxes		?
Pay after taxes		?

Question: Look at Dhruba's pay stub. Dhruba lives in a state that has state income tax. How much did Dhruba make after taxes? (unit: $)
Answer:
(Step 1) To find Dhruba's pay after taxes, subtract the total taxes from Dhruba's total earnings.
(Step 2) $620.00 - $71.14 - $48.90 - $47.00 = $452.96
(Step 3) Dhruba made $452.96 after taxes. The answer is 452.96.
Output: 452.96

Figure 10: The **correct** prediction from our PROMPTPG for a *free-text* question example. In this example, the model is asked to understand a hierarchical tax report and calculate the pay after taxes.

Table:
[TITLE] Bus schedule
the school | 8:00 A.M. | 9:15 A.M. | 9:30 A.M. | 10:00 A.M. | 11:00 A.M.
the zoo | 9:00 A.M. | 10:15 A.M. | 10:30 A.M. | 11:00 A.M. | 12:00 P.M.
the mall | 9:15 A.M. | 10:30 A.M. | 10:45 A.M. | 11:15 A.M. | 12:15 P.M.
the grocery store | 9:30 A.M. | 10:45 A.M. | 11:00 A.M. | 11:30 A.M. | 12:30 P.M.
the science museum | 10:30 A.M. | 11:45 A.M. | 12:00 P.M. | 12:30 P.M. | 1:30 P.M.
the library | 11:15 A.M. | 12:30 P.M. | 12:45 P.M. | 1:15 P.M. | 2:15 P.M.
the kickball field | 11:45 A.M. | 1:00 P.M. | 1:15 P.M. | 1:45 P.M. | 2:45 P.M.
the playground | 12:45 P.M. | 2:00 P.M. | 2:15 P.M. | 2:45 P.M. | 3:45 P.M.
the doctor's office | 1:15 P.M. | 2:30 P.M. | 2:45 P.M. | 3:15 P.M. | 4:15 P.M.

Bus schedule					
the school	8:00 A.M.	9:15 A.M.	9:30 A.M.	10:00 A.M.	11:00 A.M.
the zoo	9:00 A.M.	10:15 A.M.	10:30 A.M.	11:00 A.M.	12:00 P.M.
the mall	9:15 A.M.	10:30 A.M.	10:45 A.M.	11:15 A.M.	12:15 P.M.
the grocery store	9:30 A.M.	10:45 A.M.	11:00 A.M.	11:30 A.M.	12:30 P.M.
the science museum	10:30 A.M.	11:45 A.M.	12:00 P.M.	12:30 P.M.	1:30 P.M.
the library	11:15 A.M.	12:30 P.M.	12:45 P.M.	1:15 P.M.	2:15 P.M.
the kickball field	11:45 A.M.	1:00 P.M.	1:15 P.M.	1:45 P.M.	2:45 P.M.
the playground	12:45 P.M.	2:00 P.M.	2:15 P.M.	2:45 P.M.	3:45 P.M.
the doctor's office	1:15 P.M.	2:30 P.M.	2:45 P.M.	3:15 P.M.	4:15 P.M.

Question: Look at the following schedule. Lena is at the mall. If she wants to arrive at the library at 1.15 P.M., what time should she get on the bus?
Options: (A) 11:15 A.M. (B) 10:30 A.M. (C) 12:30 P.M. (D) 12:15 P.M.
Answer:
(Step 1) Find the mall on the schedule.
(Step 2) The mall is on the bus route at 9:15 A.M., 10:30 A.M., 10:45 A.M., 11:15 A.M., and 12:15 P.M. Find the library on the schedule.
(Step 3) The library is on the bus route at 11:15 A.M., 12:30 P.M., 12:45 P.M., 1:15 P.M., and 2:15 P.M.
(Step 4) Lena should get on the bus at 11:15 A.M. to arrive at the library at 1:15 P.M. The answer is 11:15 A.M.
Output: (A) 11:15 A.M.

Figure 11: The **correct** prediction from our PROMPTPG for a *multi-choice* question. There are 9 rows and 6 columns in the given tabular context. Our model successfully locates the target cells in the table and performs multi-hop reasoning to predict the correct answer.

Table:
science-fiction book | $4.31
mystery novel | $8.26
crossword puzzle book | $8.74
geography book | $8.61
coloring book | $8.08
paperback book | $8.45

science-fiction book	$4.31
mystery novel	$8.26
crossword puzzle book	$8.74
geography book	$8.61
coloring book	$8.08
paperback book	$8.45

Question: Ariana has $16.50. Does she have enough to buy a paperback book and a mystery novel?
Options: (A) yes (B) no
Answer:
(Step 1) To answer this question, look at the table to find the price of a paperback book and the price of a mystery novel.
(Step 2) A paperback book costs $8.45 and a mystery novel costs $8.26.
(Step 3) $8.45 + $8.26 = $16.71
(Step 4) Since $16.71 is greater than $16.50, Ariana does not have enough money to buy a paperback book and a mystery novel. The answer is no.
Output: **(B) no**

Figure 12: The **correct** prediction from our PROMPTPG for a *multi-choice* question with Boolean options. It needs to compare the budget and the total costs to verify if Ariana has enough money.

Table:
yellow tiger's eye | $0.85
piece of green slate | $0.59
piece of red sandstone | $0.19
piece of rose quartz | $0.61
smooth piece of marble | $0.45

yellow tiger's eye	$0.85
piece of green slate	$0.59
piece of red sandstone	$0.19
piece of rose quartz	$0.61
smooth piece of marble	$0.45

Question: How much money does Connor need to buy a piece of rose quartz, a piece of green slate, and a piece of red sandstone? (unit: $)
Answer:
(Step 1) Connor needs to buy a piece of rose quartz, a piece of green slate, and a piece of red sandstone. To find the total amount of money Connor needs, add the prices of the three items.
(Step 2) $0.85 + $0.59 + $0.19 = $1.63
(Step 3) Connor needs $1.63 to buy a piece of rose quartz, a piece of green slate, and a piece of red sandstone. The answer is 1.63.
Output: **1.63**
Ground truth: **1.39**

Figure 13: The **wrong** prediction from our PROMPTPG for a *free-text* question example. Our model retrieves the wrong price for the rose quartz, thus calculating the wrong cost sum of three items.

Table:
[TITLE] Apples per tree
Stem | Leaf
1 | 1, 3, 6
2 | 2, 3, 3, 6
3 | 0
4 | 3
5 | 2, 6
6 | 0, 0, 2
7 | 2, 8
8 | 4, 5, 5, 6, 7

Apples per tree	
Stem	**Leaf**
1	1, 3, 6
2	2, 3, 3, 6
3	0
4	3
5	2, 6
6	0, 0, 2
7	2, 8
8	4, 5, 5, 6, 7

Question: A farmer counted the number of apples on each tree in her orchard. How many trees have at least 51 apples but fewer than 83 apples? (unit: trees)
Answer:
(Step 1) To solve this problem, look at the table and find all the trees that have at least 51 apples but fewer than 83 apples.
(Step 2) There are three trees that have at least 51 apples but fewer than 83 apples. The answer is 3.
Output: 3
Ground truth: 7

Figure 14: The **wrong** prediction from our PromptPG for a *free-text* question example. In this example, the problem is provided with an abstract Stem-Leaf table. Our model fails to understand this domain-specific table and lacks a high-level ability of logical reasoning to get the correct answer.

Table:
[TITLE] Lawn mowers sold
Day | Number of lawn mowers
Tuesday | 8
Wednesday | 7
Thursday | 5
Friday | 7
Saturday | 9
Sunday | 8
Monday | 8

Lawn mowers sold	
Day	**Number of lawn mowers**
Tuesday	8
Wednesday	7
Thursday	5
Friday	7
Saturday	9
Sunday	8
Monday	8

Question: A garden supply store kept track of how many lawn mowers it sold in the past 7 days. What is the median of the numbers?
Answer:
(Step 1) The median is the "middle" value. To find the median, put the numbers in order from least to greatest and find the number in the middle.
(Step 2) 8, 7, 5, 7, 9, 8, 8
(Step 3) The median is 7. The answer is 7.
Output: 7
Ground truth: 8

Figure 15: The **wrong** prediction from our PromptPG for a *free-text* question example. It seems that our model has a limited ability to order numbers in the mathematical domain.

Table:
[TITLE] Tour boat schedule
Ocean City | 8:15 A.M. | 9:00 A.M. | 9:15 A.M. | 9:30 A.M. | 10:00 A.M.
Whale Watch Harbor | 9:30 A.M. | 10:15 A.M. | 10:30 A.M. | 10:45 A.M. | 11:15 A.M.
Oyster Lighthouse | 10:15 A.M. | 11:00 A.M. | 11:15 A.M. | 11:30 A.M. | 12:00 P.M.
Fisherman's Cove | 11:15 A.M. | 12:00 P.M. | 12:15 P.M. | 12:30 P.M. | 1:00 P.M.
Surfing Beach | 12:00 P.M. | 12:45 P.M. | 1:00 P.M. | 1:15 P.M. | 1:45 P.M.

Tour boat schedule					
Ocean City	8:15 A.M.	9:00 A.M.	9:15 A.M.	9:30 A.M.	10:00 A.M.
Whale Watch Harbor	9:30 A.M.	10:15 A.M.	10:30 A.M.	10:45 A.M.	11:15 A.M.
Oyster Lighthouse	10:15 A.M.	11:00 A.M.	11:15 A.M.	11:30 A.M.	12:00 P.M.
Fisherman's Cove	11:15 A.M.	12:00 P.M.	12:15 P.M.	12:30 P.M.	1:00 P.M.
Surfing Beach	12:00 P.M.	12:45 P.M.	1:00 P.M.	1:15 P.M.	1:45 P.M.

Question: Look at the following schedule. Haley is at Ocean City at 9.45 A.M. How soon can she get to Surfing Beach?
Options: (A) 1:45 P.M. (B) 12:00 P.M. (C) 10:30 A.M. (D) 12:15 P.M.
Answer:
(Step 1) Find 9:45 A.M. on the schedule.
(Step 2) The next stop is Whale Watch Harbor at 10:15 A.M.
(Step 3) The earliest she can get to Surfing Beach is 12:00 P.M. The answer is 12:00 P.M.
Output: (B) 12:00 P.M.
Ground truth: (A) 1:45 P.M.

Figure 16: The **wrong** prediction from our PROMPTPG for a *multi-choice* question example. In this example, the query of the current time provided in the question text does not hit any time slot exactly in the tabular context. Therefore, the model fails to locate the accurate time for the next stop.

Table:
[TITLE] Blocks from school
Name | Number of blocks
Wendy | 14
Carson | 8
Zach | 17
Lila | 17
Nicole | 3
Shane | 7
Henry | 5
Devin | 17

Blocks from school	
Name	**Number of blocks**
Wendy	14
Carson	8
Zach	17
Lila	17
Nicole	3
Shane	7
Henry	5
Devin	17

Question: Some students compared how many blocks they live from school. What is the mean of the numbers?
Answer:
(Step 1) The mean is the average of the numbers.
(Step 2) To find the mean, add the numbers and divide by the number of students.
(Step 3) 14 + 8 + 17 + 17 + 3 + 7 + 5 + 17 = 86
(Step 4) There are 8 students, so divide 86 by 8.
(Step 5) 86 ÷ 8 = 10.75
(Step 6) The mean is 10.75. The answer is 10.75.
Output: 10.75
Ground truth: 11

Figure 17: The **wrong** prediction from our PROMPTPG for a *free-text* question example. The example poses a challenge for the model to perform an arithmetic operation on a long chain of numbers.

Table:
[TITLE] Bus schedule
Name | Score
Anita | 128
Krysta | 135
Hugo | 137
Devin | 115

Miniature golf scores	
Name	**Score**
Anita	128
Krysta	135
Hugo	137
Devin	115

Question: Some friends played miniature golf and wrote down their scores. How many more strokes did Hugo take than Anita? (unit: strokes)
Answer:
(Step 1) Anita's score is 128. Hugo's score is 137.
(Step 2) Hugo took 9 more strokes than Anita.
Output: 137
Ground truth: 9

Figure 18: The **wrong** prediction from our PROMPTPG for a *free-text* question example. Although our model includes the correct answer in the generated output, the output does not follow the format that is designed in in-context examples. It makes our answer extractor fail to get the target answer. This issue could be alleviated by completing human-designed rules or developing an additional module to extract the answer from the prediction more accurately in various cases.